BA PROG. DSE 1(MATHS)

DIFFERENTIAL EQUATION

1) Solve the Differential equation;

a) Given that $y_1=x$, is a solution of
$(x^2+1)y'' - 2xy' + 2y = 0$, find a linearly independent solution by reducing its order.

b) Given that $y_1=x^2$, is a solution of
$(x^3-x^2)y'' - (x^3+2x^2-2x)y' + (2x^2+2x-2)y = 0$, find a linearly independent solution by reducing its order.

c) Given that $y_1=e^{2x}$, is a solution of
$(2x+1)y'' - 4(x+1)y' + 4y = 0$, find a linearly independent solution by reducing its order.

d) Find the general solution of ,
$y'' - 3y' + 2y = 4x^2$,

e) Find the general solution of ,
$y^{IV} - 4y''' + 14y'' - 20y' + 25y = 0$

f) Find the general solution of ,
$3y'' + 4y' - 4y = 0$, given that $y(0) = 2$ & $y'(0) = -4$

f) Find the general solution of ,
$y'' + 6y' + 58y = 0$, given that $y(0) = -1$ & $y'(0) = 5$

g) Find the general solution of ,
$y''' - 6y'' + 11y' - 6y = 0$, given that $y(0) = 0$, $y'(0)=0$ & $y''(0) = 2$

h) Find the general solution of ,
$y''' - 2y'' + 4y' - 8y = 0$, given that $y(0) = 2$, $y'(0)=0$ & $y''(0) = 0$

i) Find the general solution of ,
$x^2 y'' - 2xy' + 2y = x^3$,

j) Find the general solution of ,
$x^3y''' - 4x^2 y'' + 8xy' - 8y = 4\ln x$,

k) Find the general solution of ,
$x^2 y'' - 6y = \ln x$, $y(1)= 1/6$ & $y'(1)= -1/6$

l) Find the general solution of ,
$(x+2)^2y'' - (x+2)y' - 3y = 0$,

m) Find the general solution of ,
$x^2 y'' + xy' + y = 4\sin(\ln x)$,

2) Find the perticular integral of the following ODE

a) $y'' - 3y' + 2y = x^2e^x$,

b) $y'' - 2y' - 3y = 2e^x - 10\sin x$; $y(0) = 2$ & $y'(0)= 4$

c) $y'' + 2y' + 5y = 6\sin 2x + 7\cos 2x$,

d) $y'' + y = x\sin x$,

e) $y'' + 4y = 8\sin 2x$, $y(0) = 6$ & $y'(0)= 8$

f) $y'' + y = 3x^2 - 4\sin x$,$y(0)=0$ &$y'(0)=8$)

g) $(x^2+2x)y'' - 2(x+1)y' + 2y = (x+2)^2$; where CF= $C_1(x+1)+C_2x^2$

3) Use variation of parameter to find the complete solution of the following differential equation.

a) $x^3 \frac{d^3y}{dx^3} - 4x^2 \frac{d^2y}{dx^2} + 8x \frac{dy}{dx} - 8y = 4\ln x$

b) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3$

c) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 4\sin lnx$

d) $(2x-3)^2\frac{d^2y}{dx^2} - 6(2x-3)\frac{dy}{dx} + 12y = 0$

e) $(x^2+1)y'' - 2xy' + 2y = 6(x^2+1)^2$

f) $y''+y = \tan^2 x$

g) $(x+1)^2y'' - 2(x+1)y' + 2y = 1$ where CF= $C_1(x+1)+C_2(x+1)^2$

h) $(x^2+2x)y'' - 2(x+1)y' + 2y = (x+2)^2$; where CF= $C_1(x+1)+C_2x^2$

i) $x^2y'' - x(x+2)y' + (x+2)y = x^3$; where CF= $C_1x+C_2xe^x$

j) Prove that the two solutions f_1 & f_2 of the differential equation $a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$ are linear dependent iff the value of the wronskian is zero for all $a \le x \le b$, otherwise linear independent

k) Prove that if f_1 & f_2 are the two solutions of the differential equation $a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$, then $c_1f_1 + c_2f_2$ *will also be the solution. where* c_1, c_2 are the arbitrary constants.

4) Solve the equation

a. $(3x^2+4xy)dx + (2x^2 + 2y)dy = 0$
b. $(Ax^2y+2y^2)dx + (x + 4xy)dy = 0$; determine A for which the eqn is Exact & then solve it.
c. $x\sin y dx + (x^2+1)\cos y dy = 0$; $y(1) = \prod/2$
d. $(y + \sqrt{(x^2 + y^2)})dx - xdy = 0$; $y(1) = 0$
e. $(y+2)dx + y(x+4)dy = 0$ $y(-3) = -1$
f. $(x^2 + 1)dy/dx + 4xy = x$; $y(2) = 1$
g. $y^2dx + (3xy - 1)dy = 0$
h. $xy' - 2y = 2x^4$; $y(2)=8$
i. $(2x\cos y + 3x^2y)dx + (x^3 - x^2\sin y - y)dy = 0$; $y(0) = 2$
j. $(2x + \tan y)dx + (x - x^2\tan y)dy = 0$
k. $(8x^2y^3 - 2y^4)dx + (5x^3y^2 - 8xy^3)dy = 0$

5) Differential equation solvable for p, x & y

a. $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$
b. $p^2 + 2pyCotx = y^2$
c. $y^2 + xyp - x^2p^2 = 0$
d. $y = 2xp - xp^2$
e. $y + px = x^4p^2$
f. $y = 2px + y^2p^3$
g. $p^3y^2 - 2xp + y = 0$
h. $x^2(y - px) = yp^2$
i. $4y = x^2 + p^2$

6) Solving simultaneous equation;

a. $4\frac{dx}{dt} + 9\frac{dy}{dt} + 44x + 49y = t$

$$3\frac{dx}{dt} + 7\frac{dy}{dt} + 34x + 38y = e^t$$

b. $\frac{dx}{dt} + 2\frac{dy}{dt} - 2x + 2y = 3e^t$

$$3\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 4e^{2t}$$

c. $\frac{dx}{dt} + 4x + 3y = t$

$\frac{dy}{dt} + 2x + 5y = e^t$

d. $2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = t$

$2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y = 2$

e. $\frac{d^2x}{dt^2} + 4\frac{dy}{dt} + x - 4y = 0$

$\frac{dx}{dt} + \frac{dy}{dt} - x + 9y = e^{2t}$

f. $\frac{dx}{z-y} = \frac{dy}{x-z} = \frac{dz}{y-x}$

g. $\frac{adx}{(b-c)zy} = \frac{bdy}{(c-a)xz} = \frac{cdz}{(a-b)yx}$

h. $\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}$

i. $\frac{dx}{z+y} = \frac{dy}{x+z} = \frac{dz}{y+x}$

j. $\frac{dx}{x(y^2-z^2)} = \frac{dy}{-y(z^2+x^2)} = \frac{dz}{z(x^2+y^2)}$

7) Writing Partial differential equation;

a. $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$

b. $2z = (ax + y)^2 + b$

c. $lx + my + nz = f(x^2 + y^2 + z^2)$

d. $f(x + y + z, xyz) = 0$

e. Find the partial differential equation of all spheres of radius a having their centers in the xy – plane. [Hint :$(x-\alpha)^2 + (y-\beta)^2 + z^2 = a^2$]

f. $ax^2 + by^2 + z^2 = 1$

8) Find the general solution of the given PDE;

a. $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$

b. $px(x + y) = qy(x + y) - (x - y)(2x + 2y + z)$

c. $(y + zx)p - (x + yz)q = x^2 - y^2$

d. $x(x^2 + 3y^2)p - y(3x^2 + y^2)q = 2z(y^2 - x^2)$

e. $(y - z)\frac{\delta u}{\delta x} + (z - x)\frac{\delta u}{\delta y} + (x - y)\frac{\delta u}{\delta z} = 0$ Show that u contains x, y, z only in combination x+y+z & $x^2 + y^2 + z^2$.

f. $z(xp - yq) = y^2 - x^2$

g. $x^2\frac{\delta z}{\delta x} + y^2\frac{\delta z}{\delta y} = (x + y)z$

9) Compatibility and general integral;

a. Show that the equations $f(x, y, z, p, q) = 0$ & $g(x, y, z, p, q) = 0$ $are\ compatible\ if$ $\frac{\delta(f,g)}{\delta(x,p)} + \frac{\delta(f,g)}{\delta(y,q)} = 0$

b. $xp - yq = x, x^2p + q = xz.$ Prove that this is compatible and their solution.

c. $xp = yq,\ z(xp + yq) = 2xy.$ Prove that this is compatible and their solution.

d. Show that the equations: $p^2 + q^2 = 1$ & $(p^2 + q^2)x = pz$ are compatible and solve them.

10) Complete Integral (Charpit's Method);

a. $z = pq$

b. $p = (z + qy)^2$

c. $z^2 = pqxy$

d. $(p^2 + q^2)y = qz$

e. $(p^2 + q^2) = 1$

f. $(p + q) = pq$

g. $(x^2 + y^2)(p^2 + q^2) = 1$

h. $p^2 y(1 + x^2) = qx^2$

11) Canonical form (Reduce the equation in canonical form) ;

a. $\frac{\delta^2 z}{\delta x^2} - \frac{\delta^2 z}{\delta y^2} = 0$

b. $\frac{\delta^2 z}{\delta x^2} - x^2 \frac{\delta^2 z}{\delta y^2} = 0$

c. $(n-1)^2 \frac{\delta^2 z}{\delta x^2} - y^{2n} \frac{\delta^2 z}{\delta y^2} = n y^{2n-1} \frac{\delta z}{\delta y}$

d. $\frac{\delta^2 z}{\delta x^2} - \frac{1}{c^2} \frac{\delta^2 z}{\delta t^2} = 0 \; ; c > 0$

e. $\frac{\delta^2 z}{\delta x^2} + x^2 \frac{\delta^2 z}{\delta y^2} = 0$

Good Luck!!!!